Communication

→ communication is the process of establishing connection or link between two points for information exchange.

(or)

→ communication is simply the process of conveying message at a distance or communication is the basic process of exchanging information.

→ The electronic equipments which are used for communication purpose are called communication equipments or elements.

Communication System is the combination of ~~all~~ different communication elements.

e.g :- The radio, & Television, line telephony, line telegraphy etc.

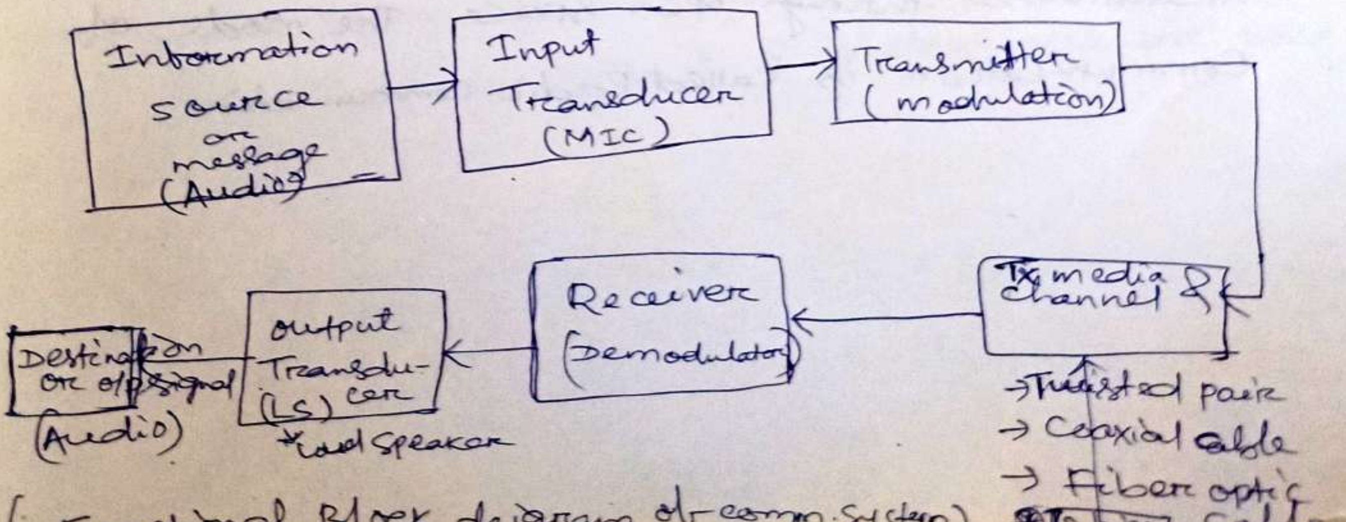
~~Information Source or message~~

Voice → 30~~Hz~~ 3.5 KHZ Telephone

Audio → 20 - 20 KHZ Broadcasting

Video → 0 - 4.5 MHZ TV

data → freq. depends upon pulse width Internet



(Functional Block daigram of comm. system)

→ Source originates message. message may be voice [illegible] (or Audio) signal, video signal or data.

→ If message is non-electrical form, it must be converted by an input Transducer into an electrical wave form. ~~Called message signal~~

e.g:- Microphone is an input Transducer which converts the information (Audio signal) into electrical signal.

→ Transmitter then modifies it for better transmission (modulation)

→ channel is the medium for transmission.

It can be
BW
Twisted pair (KHZ)
Coaxial cable (MHZ)
Fibber optic cable (GHZ)
} Wire line channels

or air (or) freespace → wireless channel.

→ Receiver reconstructs the received signal from channel & sends to output Transducer.

→ output Transducer converts the received electrical signal to original form.
e.g:- Loud speaker

→ In wireless, message is converted into electrical form by Transducer & then converted into electromagnetic waves (i.e. Called Radio waves) by Transmitter & transmitted through open space. The mode of communication is called Radio communication.

<3KHz ~~ELF → underwater~~

Communication channel & their characteristics

① **Wireline channels** :- The channel which is wire medium through which voice signal, data, video signal can be transmitted.

e.g :- Twisted pair ~~wire~~ wireline and coaxial cable.

→ Signals transmitted through such channels are distorted in both amplitude and phase and further corrupted by additive noise.

→ Twisted pair wireline channels are also prone to (subject to) crosstalk interference ~~from~~ physically adjacent channels.

→ So, to compensate it channel equalizers are used.

② **Fiber optic channels** :-

→ In this, low signal attenuation & distortion.

→ Bandwidth is very high.

→ The Transmitter or modulator in a Fiber optic Communication system is a light source, either a LED or a laser. Information is transmitted by varying (modulating) the intensity of the light source with the message signal & photodiodes are used as Receiver.

③ wireless electromagnetic channels :

→ In radio communication systems, electromagnetic signals are propagated in wireless medium.

↓ wavelength Long Wave radio	< 3 KHz	ELF	→ underwater comm: (wave length)
	3 KHz to 30 KHz	VLF	Navigation 100Km – 10Km (10^5m – 10^4m)
	30KHz to 300KHz	LF	→ 10Km – 1Km (10^4m – 10^3m)
Short waves radio	300KHz to 3MHz	MF	→ AM radio × 10^3m – 10^2m Radio Broadcast
	3MHz to 30MHz	HF	→ (10^2m – 10m) → short wave radio ×
	30MHz to 300MHz	VHF	→ (10m – 1m) TV Broadcast
↑	300MHz to 1GHz	UHF	→ (1m – 0.1m) (1m – 10cm)

Microwave radio	1 GHz to 2GHz	L band → ATC Radar
	2GHz to 4HZ	S band → TV broadcast mobile cellular comm
	4GHz to 8GHz	C band → Satellite comm
	8 GHz to 12GHz	X band → laboratory (experimentation)
	12GHz to 18GHz	Ku band → Satellite broadcasting
Radar, Satellite & military	18GHz to 24GHz	K band
	24GHz to 30 GHz	Ka band (Centimeter waves band)
	30 GHz to 300GHz	millimeter wave band
	> 300 GHz	Submillimeter wave band

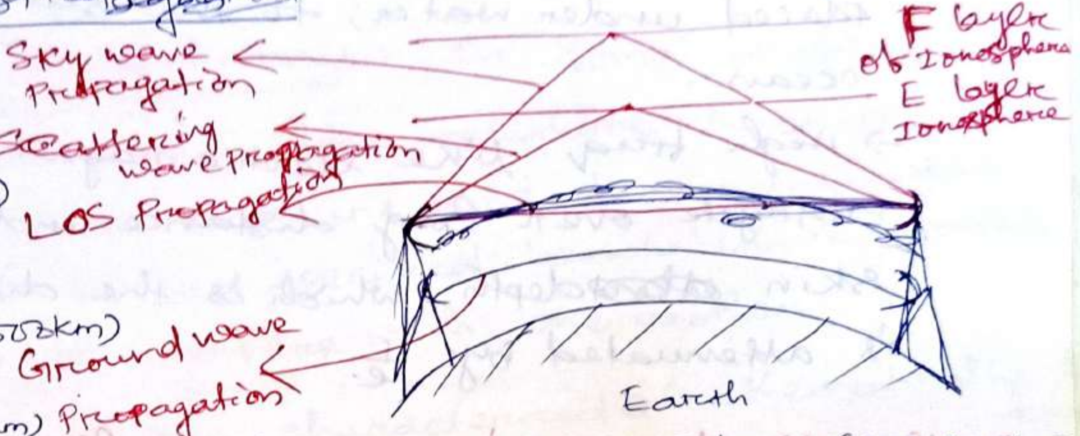
(Frequency range for wireless electromagnetic channels)

Types of propagation

① Ground wave propagation
② Sky-wave Propagation
③ Space wave Propagation (LOS)
④ Scattering wave Propagation .

~~Ground wave propagation~~

Diff. layers of Earth
① Troposphere
(up to above 18km)
② Stratosphere
(18 - 50km)
③ Ionosphere (50-500km)
④ Exosphere
(above 500km)
→ (a) D-layer (50-90km) (b) E-layer (90-150km) (c) F-layer (150-500km)



Ground wave propagation

→ In this mode, the signals travels very close to the surface of earth.

→ The ground wave follows the curvature of earth.

EX :- Telephone communication

Sky-wave propagation

→ In this mode, the waves are Propagated ~~reflected back~~ from transmitting antenna to receiving antenna through F-layer of Ionosphere

EX :- long distance Radio communication.

Line of Sight (LOS) or Space wave Propagation

→ In this, the wave Propagates from Transmitter to Receiver in direct or straight path.

EX :- TV transmission.

Scattering wave propagation

→ In this mode, signal transmitted from Tx to Rx through E-layer of Ionosphere.

④ Under water Acoustic channel :

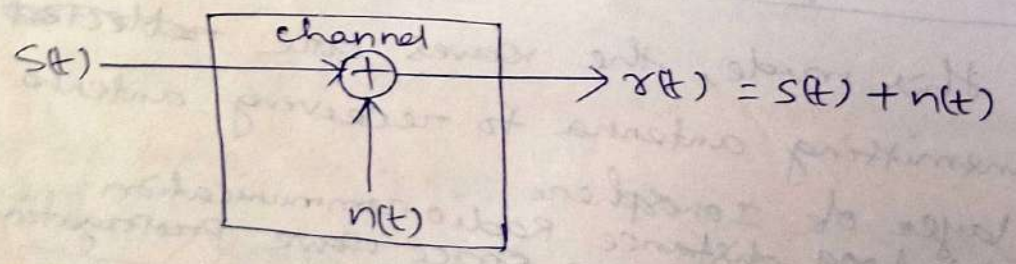
Datas are transmitted and collected by sensors placed under water, to the surface of ~~water~~ the ocean.

→ High freq. like electromagnetic wave don't propagate over long distance under water due to skin ~~effect~~ depth, which is the distance a signal is attenuated by $\frac{1}{e}$.

Mathematical model for communication channels

Design of communication systems for transmitting information through physical channels is characterized by a mathematical model that reflect the most important characteristics of the transmission medium.

(1) Additive noise channel :- It is the simplest mathematical model.



s(t) ⇒ Tramitted signal

n(t) ⇒ additive noise signal

r(t) → received signal.

- The additive noise signal is arised from electronic components. which is normally Gaussian type.

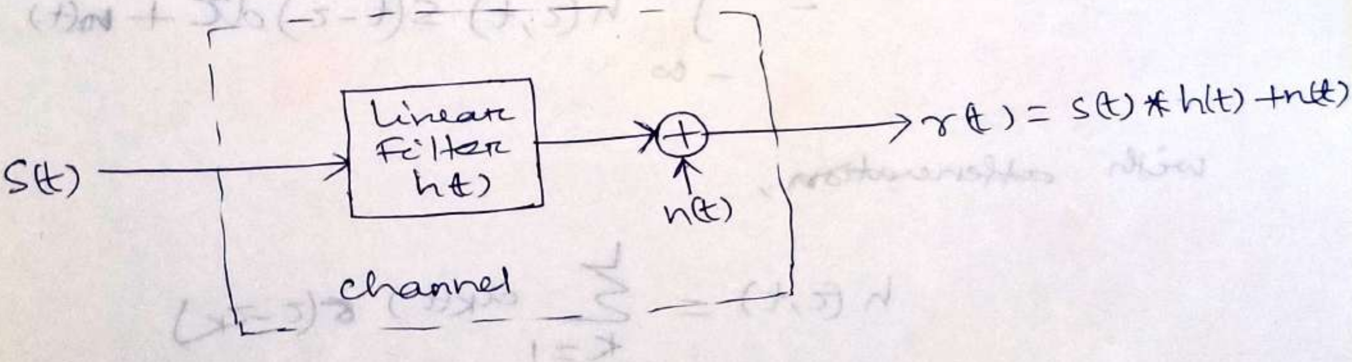
✓ → If the signal undergoes channel attenuation, then received signal

$$r(t) = as(t) + n(t)$$

where a → attenuation factor.

(2) ~~The~~ <u>Linear Filter channel</u> :- In some physical channels such as wireline telephone channels, filters are used to ensure that the transmitted signals do not exceed specified bandwidth limitations and thus do not interfere with one another.

such channel are characterized mathematically as linear filter chanels with additive noise.



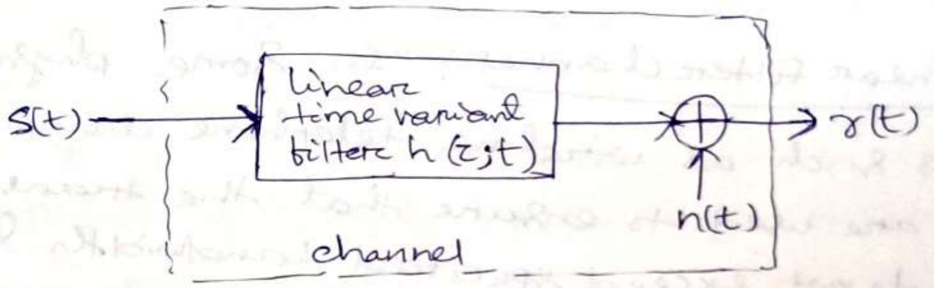
$$r(t) = \underbrace{s(t) * h(t)}_{\text{convolution.}} + n(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, s(t-\tau)\, d\tau + n(t)$$

where h(t) is the impulse response of the filter.

③ linear time variant filter channel

Physical channels such as under water acoustic channels and ionospheric radio channels which result in time variant multipath propagation of the transmitted signal is characterized mathematically as time-variant linear filters.



linear time variant filter response is $h(\tau,t)$

$$r(t) = s(t) * h(\tau,t) + n(t)$$

$$= \int_{-\infty}^{\infty} h(\tau,t)\, s(t-\tau)\, d\tau + n(t)$$

with attenuation,

$$h(\tau,t) = \sum_{k=1}^{L} a_k(t)\, \delta(\tau-\tau_k)$$

Where $a_k(t) \rightarrow$ time variant attenuation ~~factors~~ factor for the L multipath propagation paths.

Now, $$r(t) = \sum_{K=1}^{L} a_K(t)\, s(t-\tau_K) + n(t)$$

$$\boxed{\because \int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = x(t_0);\ t_1<t_0<t_2}$$

Frequency Domain Analysis of Signals and Systems.

Signals :- Anything that bears information can be Considered as signal.

e.g :- speech, music, video, speed of an automobile etc.

→ Mathematically signals are modeled as function of one or more independent variables.

Examples of independent variable used to represent signals are time, frequency or spatial coordinates.

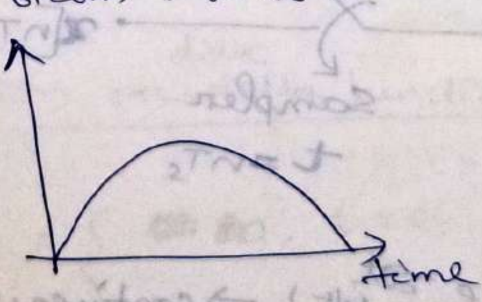
→ To extract required information from a set of signals, processing is done by electronic systems, so the signal must first be converted into an electric signal that is a voltage or a current.

Transducer :- which converts one form of energy to another.

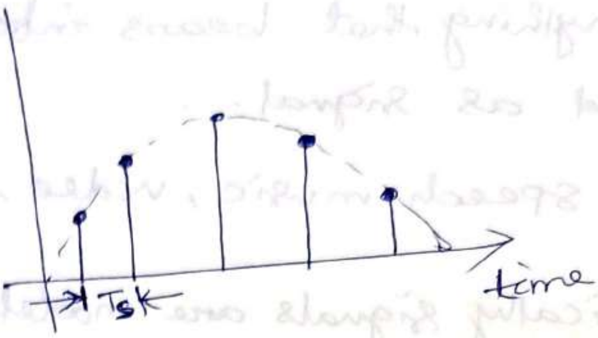
Types of signals

① **Contineous time or Analog signals** :- contineous both in time and amplitude. Contineous in amplitude means that the amplitude can assume any value in the contineous range from $-\infty$ to $+\infty$.

Analog

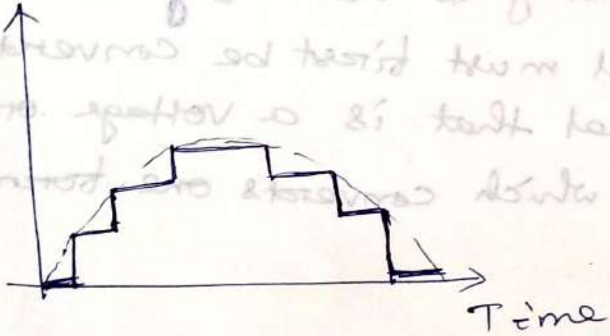


② Discrete time signals :- Discrete in time and continueous in amplitude. (integere values of time.

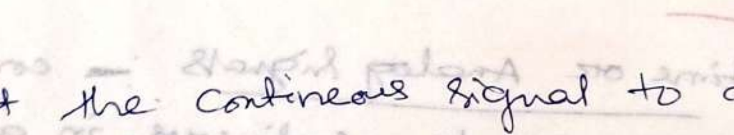


③ Digital signals :- Discrete both in time and amplitude

→ If the amplitude of a signal can assume only a value from a finite set of numbers, the amplitude is said to be discretized or quantized.



To convert the contineous signal to discrete signal, we use sampler.

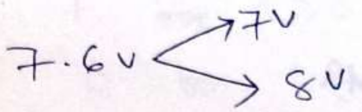


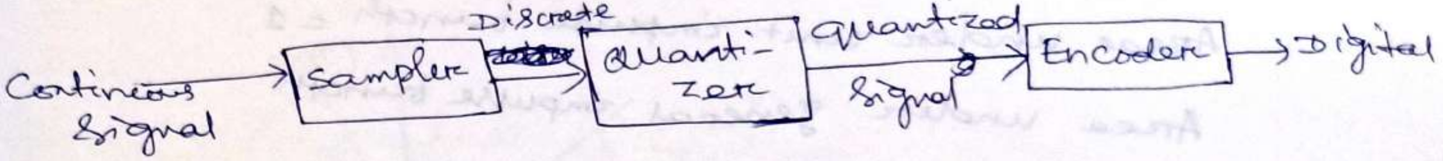
$x(t) = e^{-2t} u(t) \rightarrow$ contineous signal

$$= e^{-2nT_s} u[nT_s]$$

$f_s \geq 2f_m$ $\qquad$ $T_s = 1 sec.$

Quantization :- Converting contineous amplitude to discrete amplitude is known as quantization.



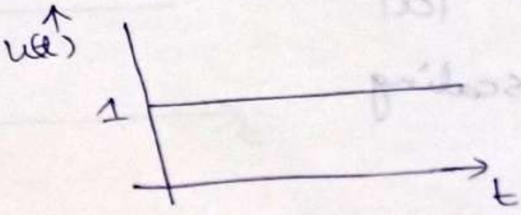


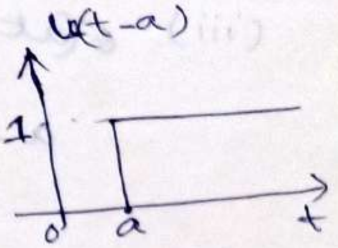
Standard signals (functions)

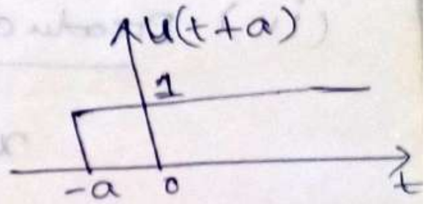
① unit step function [u(t)]

$u(t) = 0 \ ; \ t<0$

$\quad = 1 \ ; \ t>0$







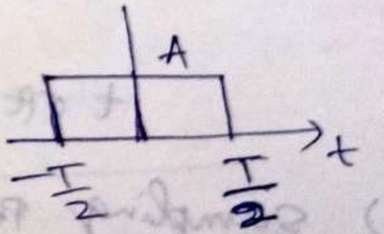
② Rectangular (or) Gate function

$A\,\text{rect}(t/T)$ or $A\Pi(t/T)$

$$g(t) = \left\{ A \ ; \ -\frac{T}{2} < t < \frac{T}{2} \right.$$

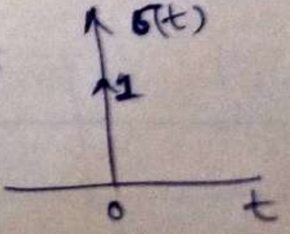
A → Amplitude

T → width



③ Contineous time Impulse function or dirac delta function

$$\delta(t) = \begin{cases} \infty \ ; & t = 0 \\ 0 \ ; & t \neq 0 \end{cases}$$



unit impulse, $\delta(t) = 1 \ ; \ t = 0$

(i) $$\underbrace{\int_{-\infty}^{\infty} \delta(t)\, dt = 1}_{\text{Area/Strength}}$$

Areas under unit impulse functn = 1

Area under general impulse functn

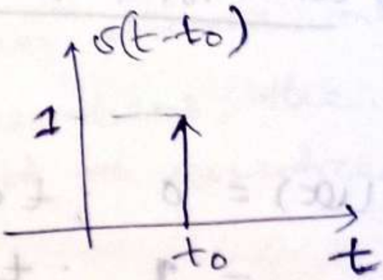
$$\int_{-\infty}^{\infty} A\,\delta(t)\, dt = A$$

(ii) $\delta(-t) = \delta(t)$

(iii) $\delta(at) = \frac{1}{|a|}\delta(t)$

$a \to$ scaling

$\underline{\delta(t-t_0)}$



(iv) Product Property

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

e.g:- $\sin t\, \delta\left(t-\frac{\pi}{4}\right)$

$$= \sin\left(\frac{\pi}{4}\right) \cdot \delta\left(t-\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}}\, \delta\left(t-\frac{\pi}{4}\right)$$

$$t\,\delta(t) = \underline{0}$$

(V) Sampling Property

$$\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = x(t_0) \; ; \; t_1 < t_0 < t_2$$

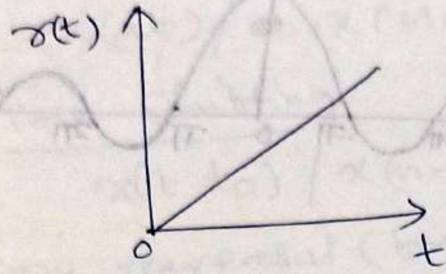
$$\int_{-1}^{2} (t+t^2)\,\delta(t-4)\,dt = 0$$

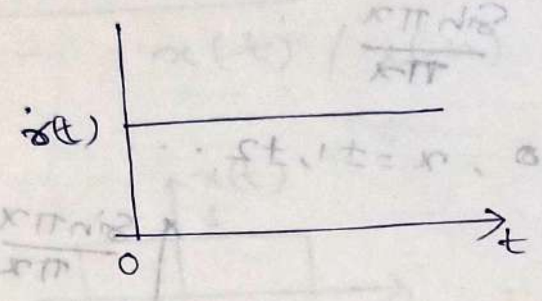
$$\int_{0}^{\infty} (t+\cos\pi t)\,\delta(t-1)\,dt$$

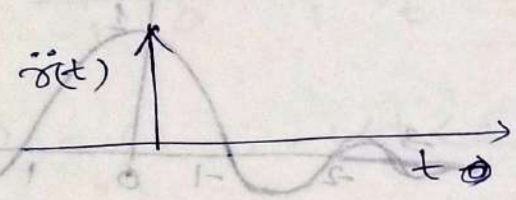
$$= 1 + \cos\pi = 0$$

Ramp function

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$



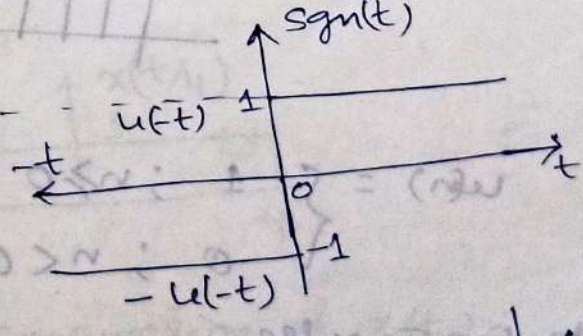




$$\delta(t) = \frac{d}{dt} u(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(t)\, dt$$

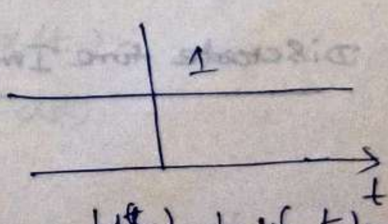
Signum function [sgn(t)]



$$\mathrm{sgn}(t) = u(t) - u(-t)$$

$$= u(t) - [1 - u(t)]$$

$$= 2u(t) - 1$$

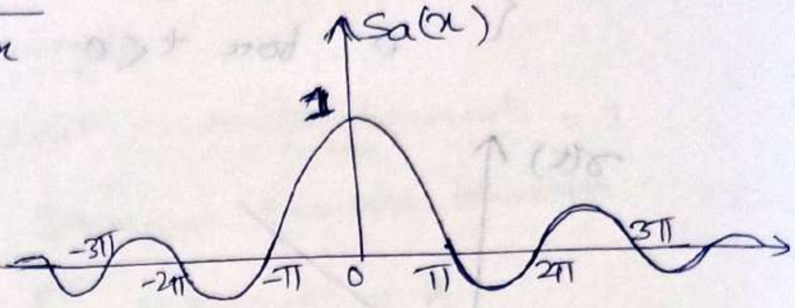


$u(t) + u(-t) = 1$

$u(-t) = 1 - u(t)$

Sampling function

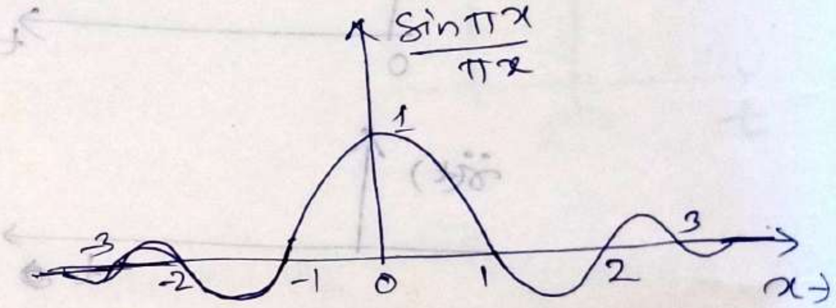
$$Sa(x) = \frac{\sin x}{x}$$



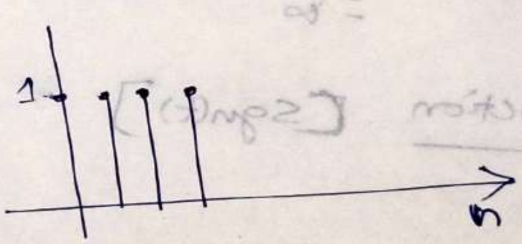
another form is

$$\text{Sinc}\,x = \frac{\sin \pi x}{\pi x}$$

$$= 0 \ , \ x = \pm 1, \pm 2 \cdots$$



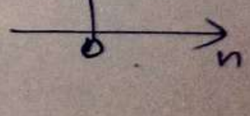
Unit step sequence



$$u(n) = \begin{cases} 1 \ ; n \geq 0 \\ 0 \ ; n < 0 \end{cases}$$

Discrete time Impulse

$$\sigma(n) = \begin{cases} 1 \ ; n = 0 \\ 0 \ ; n \neq 0 \end{cases}$$



Transformations of a signal

we have 3 transformations:

① Time scaling

$x(at)$ / $x(Mn)$

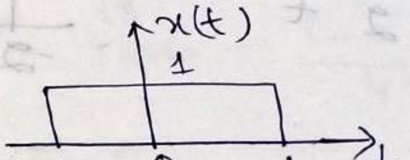
② Time shifting

$x(t-t_0)$ / $x(n-n_0)$

③ Time reversal (folding)

$x(-t)$ / $x(-n)$

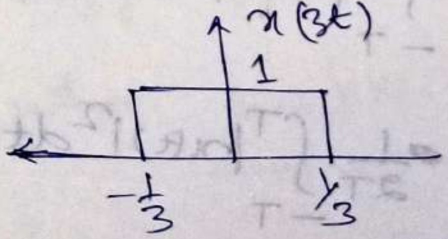
①



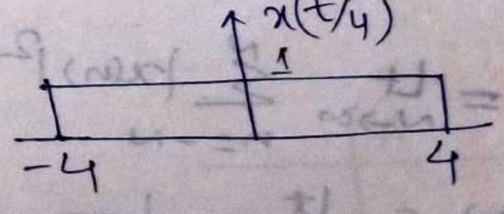
$x(t) = \{1 \;;\; -1<t<1$

$x(3t) = \{1 \;;\; -1 < 3t < 1$

$-\frac{1}{3} < t < \frac{1}{3}$



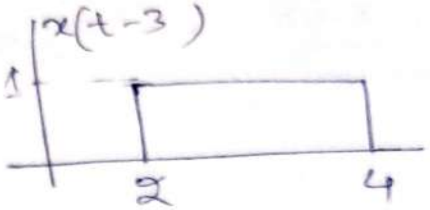
$x(t/4) \rightarrow$



If $a>1$ → compression of $x(t)$

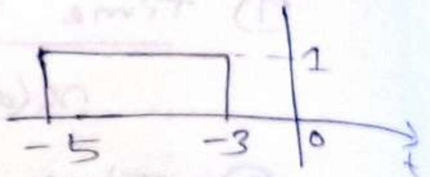
If $a<1$ → Expansion of $x(t)$

② $x(t-3)$ $\quad t_0 = 3$ (right shift)

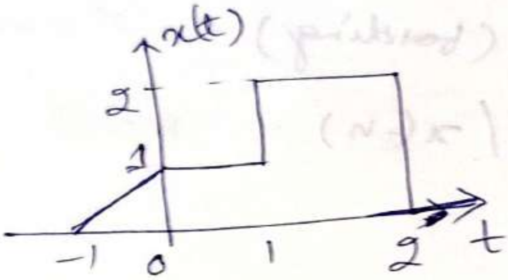


$x(t+4)$

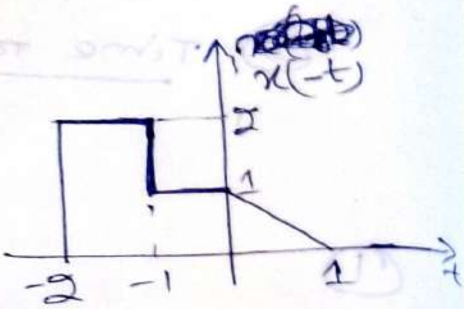
$t_0 = -4$ (Left shift)



③



→



classification of signals

① Energy and power signal

$\downarrow$ $0 < E < \infty$ $\qquad$ $\downarrow$ $0 < P_{av} < \infty$

$$E_{x(t)} = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2 \, dt$$

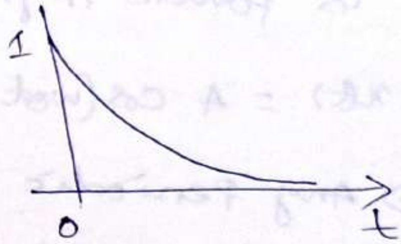
$$P_{av}\, x(t) = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$E_{x(n)} = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} |x(n)|^2$$

$$P_{av}\, x(n) = \underset{N\to\infty}{Lt} \frac{1}{2N+\underset{\overset{\downarrow}{n=0}}{\textcircled{1}}} \sum_{n=-N}^{N} |x(n)|^2$$

Q// Find wether the following signals are energy or power signal .

① $x(t) = e^{-t}u(t)$



$$P_{av} = \underset{T\to\infty}{lt} \frac{1}{2T}\int_{0}^{T} e^{-2t}\,dt$$

$$= \underset{T\to\infty}{lt} \frac{1}{2T}\left[\frac{e^{-2t}}{-2}\right]_{0}^{T}$$

$$= \underset{T\to\infty}{lt} \frac{1-e^{-2T}}{4T}$$

$$= 0$$

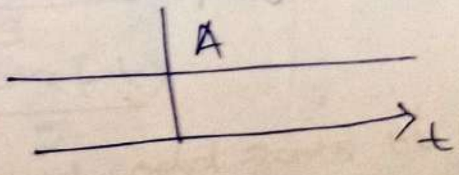
$$E = \underset{T\to\infty}{lt} \frac{1-e^{-2T}}{2} = \frac{1}{2}$$

So, energy signal.

when energy signal, avg. power is zero.

$$\left.\begin{matrix} \text{when } t\to\pm\infty \\ amp\to 0 \end{matrix}\right] \text{energy signal}$$

② $x(t) = A$



$$P_{av} = \underset{T\to\infty}{lt} \frac{1}{2T}\int_{-T}^{T} A^2\,dt$$

$$= A^2$$

$$E = \infty$$

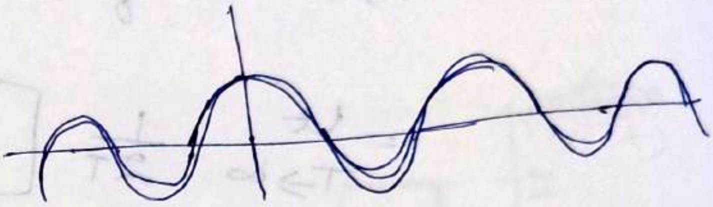
A power signal has infinite energy.

A signal which takes const. amplitude from $-\infty$ to $+\infty$ is power signal.

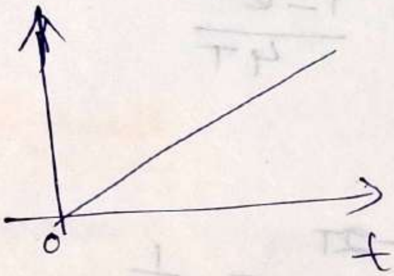
③ $x(t) = A\cos(\omega_0 t + \theta)$

→ Any periodic signal is a power signal.

$$Pow = \frac{A^2}{2}$$

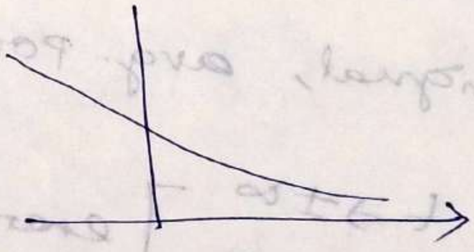


④ $x(t) = t\,u(t)$



$\left.\begin{array}{l} t \to \infty \\ amp \to \infty \end{array}\right]$ Neither energy nor Power signal.

⑤ $x(t) = e^{-t}$

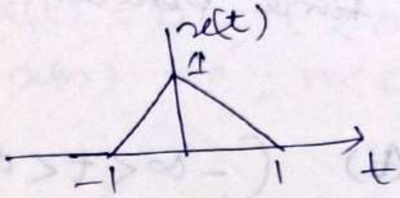


$\left.\begin{array}{l} As\ t \to -\infty \\ amp \to \infty \end{array}\right]$ Neither energy nor Power signal.

② Even and odd signals

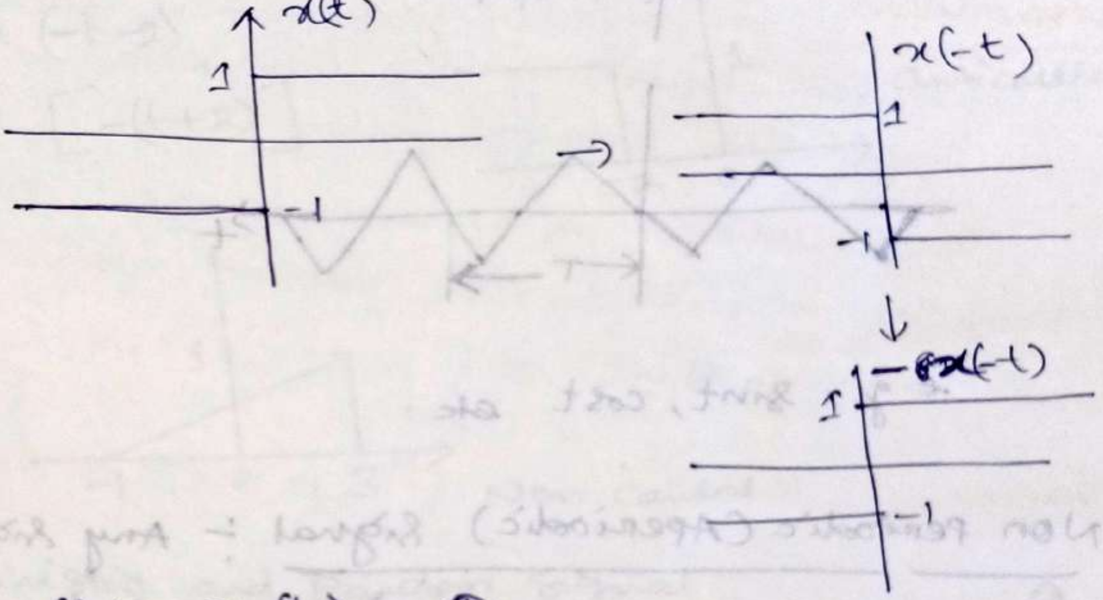
$$x(t) = x(-t) \rightarrow \text{Even}$$

e.g :-



odd

$$x(t) = -x(-t)$$



$$x(t) = x_e(t) + x_o(t) \quad — ①$$

$$x(-t) = x_e(-t) + x_o(-t)$$

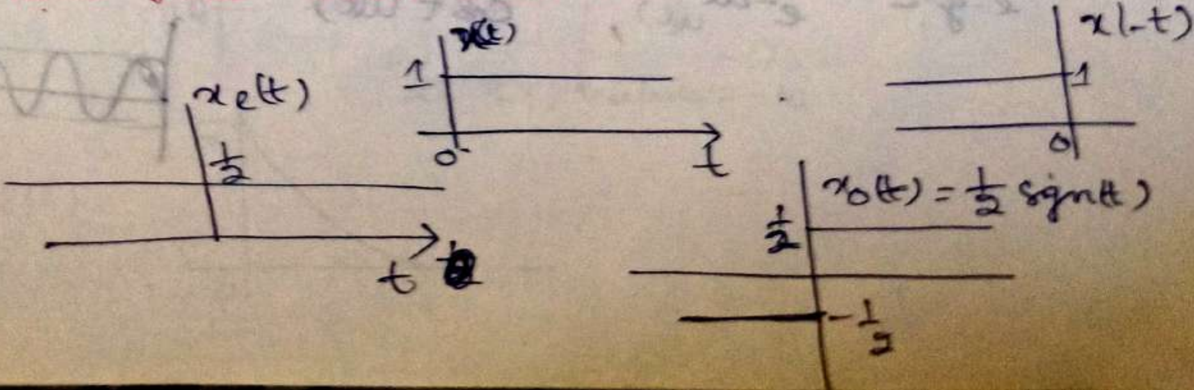
$$= x_e(t) - x_o(t) \quad —(2)$$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

✓Q11 what are the even and odd parts of u(t) ?

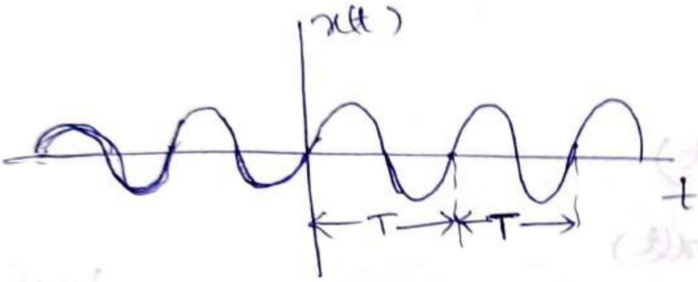
ans :-

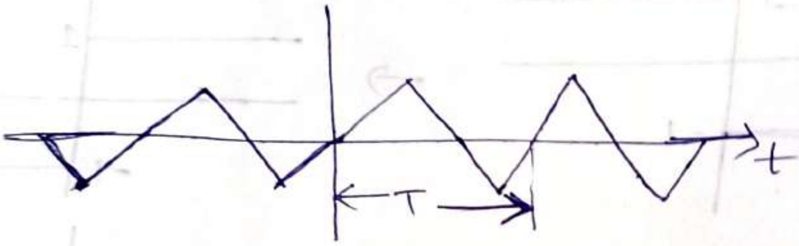


③ Periodic and non periodic signal

Periodic signal :- Any signal which has specific pattern and repeats over with time period (T) is called periodic signal.

i.e. $x(t+T) = x(t)$ $(-\infty < t < \infty)$

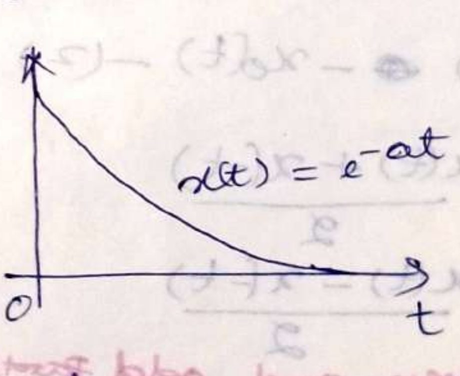




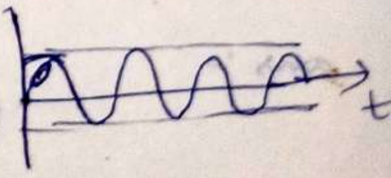
e.g. :- sint, cost etc.

Non periodic (Aperiodic) signal :- Any signal which does not repeat over with time period.

i.e. $x(t+T) \neq x(t)$ $(-\infty < t < \infty)$



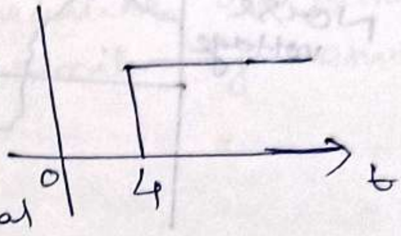
e.g. :- $e^{-at}u(t)$, $\cos t\, u(t)$



(4) Causal & noncausal signals :

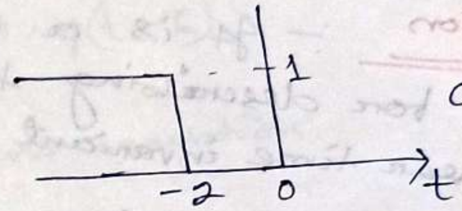
A signal is causal if it exist for $t > 0$ or $n \geq 0$

$$x(t) = 0 \; ; \; t < 0$$
$$x(n) = 0 \; ; \; n < 0$$

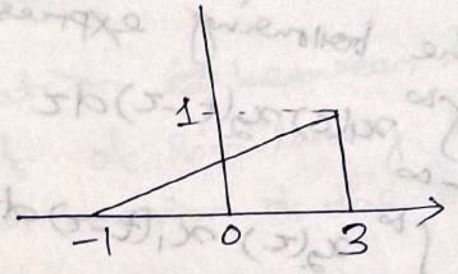


Causal signal

$$u(-t-2)$$
$$= u\left[-(t+2)\right]$$



anticausal



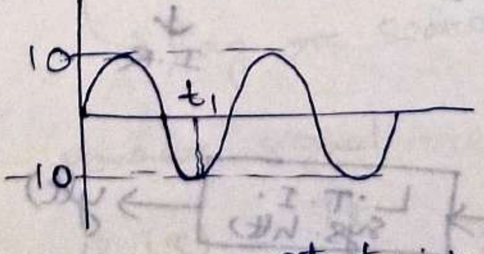
Non causal

(5) Deterministic and Random signal

①

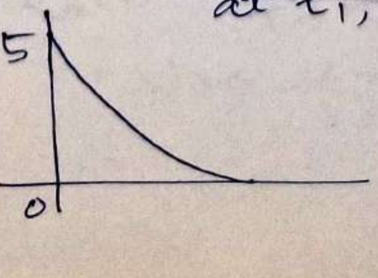
→ Deterministic signal is one where there is no uncertainity in it's value. ~~or which is represented either mathematically or graphically~~. All deterministic signal can be represented by mathematically .

e-g:- (i)

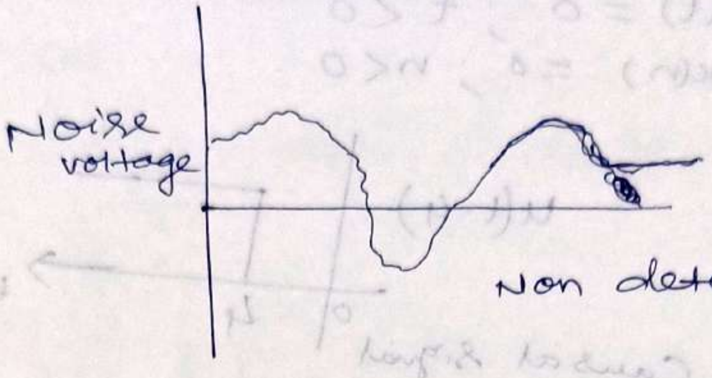


at t_1; value = -10

(ii)



→ A Random signal is a signal for which there exist some degree of uncertainity before its actual occurance. It can't be expressed mathematically.



Non deterministic signal.

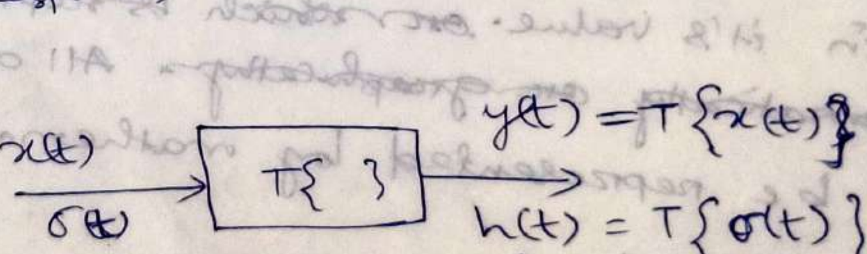
Convolution :- It is a mathematical operation and is useful for describing the input/output relationship in a linear time invariant system.

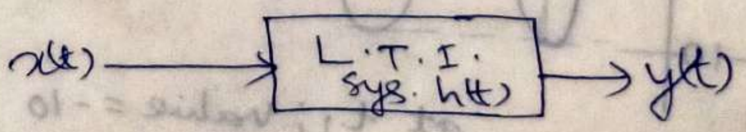
The convolution of two time functions $x_1(t)$ and $x_2(t)$ may be defined by the following expression

$$x_1(t) \circledast x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

$$\text{or} \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau)\, d\tau$$

→ An L.T.I system is always consider with respect to impulse response (if the i/p is impulse, the o/p is impulse response).



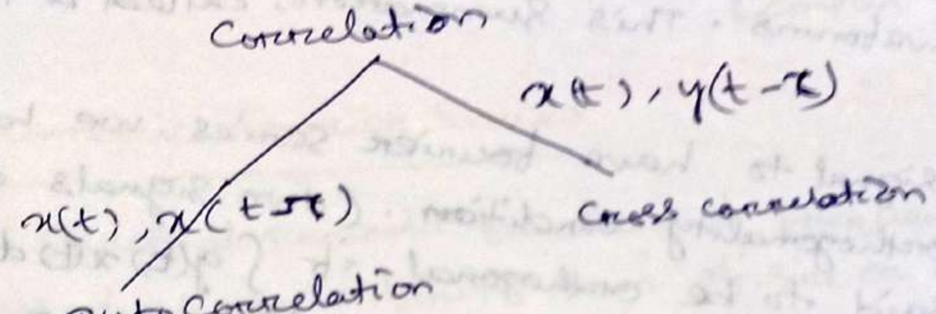


$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

or $$y(t) = \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$$

Correlation :- It is the measure of similarity between two functions/signals or similarity between the same signals



Cross correlation between two waveforms is the measure of of similarity between one waveform and time delayed version of the other waveform ②

A.C.F. of an energy signal $x(t)$

$$R_x(\tau) = \int_{-\infty}^{+\infty} x(t)\, x(t-\tau)\, dt$$

$$= \int_{-\infty}^{\infty} x(t+\tau)\, x(t)\, dt$$

→ lag or searching Parameter

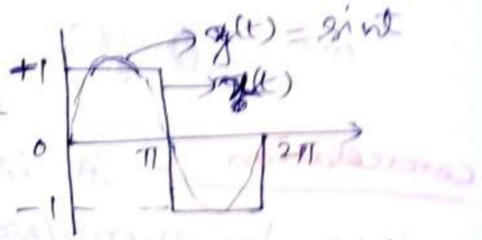
A.C.F of Power signal

$$R_x(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, x(t-\tau)\, dt$$

Fourier Series :-

It is an approximation Process in which non-sinusoidal waveform is converted to sinusoidal waveform.

$$x(t) \simeq c g(t) ; t_1<t<t_2$$



→ A periodic function of time x(t) having fundamental Period T_0 can be ~~represented~~ represented as an infinite sum of sinusoidal waveforms. This summation, called a Fourier series.

→ For any signal to have fourier series, we have to satisfy orthogonality condition. (two signals g(t) and x(t) are said to be orthogonal if $\int g(t) x(t) dt = 0$). This satisfied by sinusoidal & exponential. so we take these only for fourier series.

Dirichlet's conditions

- There are sufficient ~~sufficient~~ conditions that need to be satisfied by a function x(t) for its fourier series representation. These conditions are known as Dirichlet's conditions.

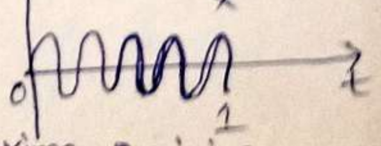
① x(t) is absolutely integrable over its Period

i.e. $\int_0^{T_0} |x(t)| dt < \infty$

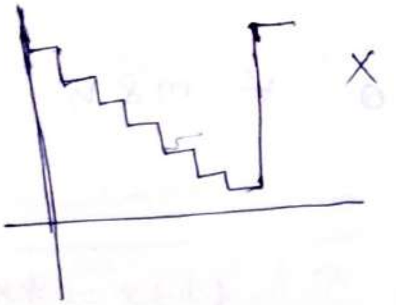
② The number of maxima & minima of x(t) in each period is finite.

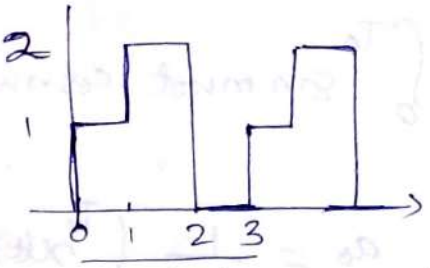
③ The no. of discontinuities of x(t) in each period is finite.

$$x(t) = \sin\left(\frac{2\pi}{t}\right) \quad 0<t<1$$



infinite maxima, minima





T = 3 finite = 3 or 4 not more

Fourier Series

Trigonometric F.S.

Exponential F.S. (Complex F.S.)

Trigonometric Fourier Series

Any Practical periodic function of frequency wo can be expressed as an infinite sum of sine or cosine functions that are integral multiples of wo.

$$x(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \cdots + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \cdots$$

⑥

$$x(t) = \underset{\downarrow \atop d.c.}{a_0} + \underbrace{\sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)}_{\downarrow \atop a.c.}$$

$\omega_0 \rightarrow$ fundamental freq.

$a_0, a_n, b_n \rightarrow$ T.F.S. coefficients.

$n\omega_0 \rightarrow$ nth harmonic of ω_0.

later of ao, an, bn

$$* \boxed{\int_0^{T_0} \cos n\omega_0 t = \int_0^{T_0} \sin n\omega_0 t = 0}$$

$$\int_0^{T_0} \cos m\omega_0 t \cdot \cos n\omega_0 t \;\; \& \;\; \int_0^{T_0} \sin m\omega_0 t \cdot \sin n\omega_0 t = \begin{cases} \frac{T_0}{2} & ; m = n \\ 0 & ; m \neq n \end{cases}$$

$$\int_0^{T_0} \sin m\omega_0 t \cdot \cos n\omega_0 t = 0 \quad \forall \; m \,\&\, n$$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\, dt$$

$\downarrow$
d.c. or avg. value

$$\left(\omega_0 = \frac{2\pi}{T_0}\right)$$

$$a_n = \frac{2}{T_0}\int_0^{T_0} x(t) \cdot \cos n\omega_0 t \, dt$$

$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t) \sin n\omega_0 t \, dt$$

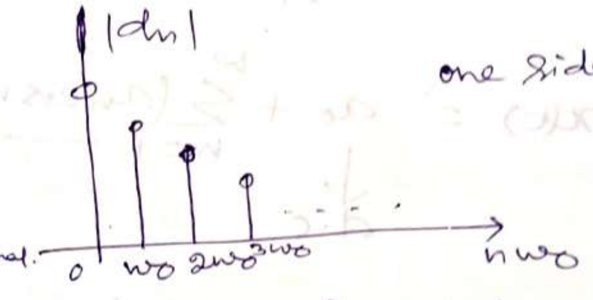
Polar form (Compact form) of T.F.S.

$$x(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos(n\omega_0 t - \phi_n)$$

where $d_0 = a_0$

$|d_n| = \sqrt{a_n^2 + b_n^2}$ → magnitude spectrum

$\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$ → phase spectrum

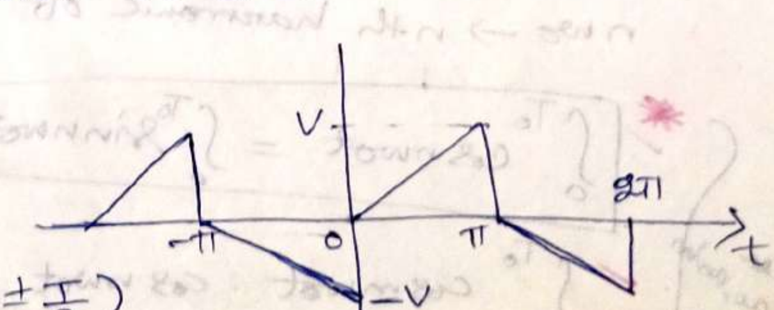


one sided spectrum

Fourier series gives discrete form of signal whereas Fourier Transform gives Continuous Form of Signal.

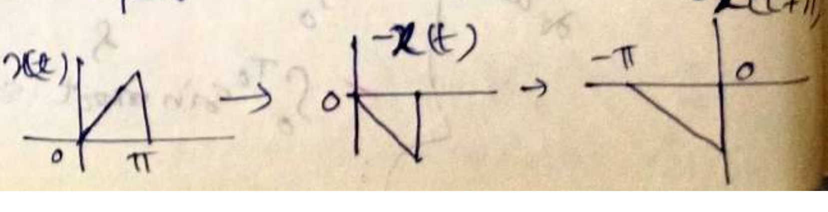
Spectrum = It provides an identity that no other signal has frequency. ~~or diff. freq~~ or variation of freq. w.r.t. time.

Half wave



$T = 2\pi$

$$x(t) = -x\left(t \pm \frac{T}{2}\right)$$
$$= -x(t \pm \pi)$$



Effect of symmetry on Fourier coefficients

Symmetry	Condition	a_0	a_n	b_n
Even	$x(t) = x(-t)$	?	?	0
odd	$x(t) = -x(-t)$	0	0	?
Half wave	$x(t) = -x(t \pm \frac{T}{2})$	0	$= 0$; n even $= ?$ n odd	$= 0$; n even $= ?$; n odd

Exponential F.S. or (complex F.S.)

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n w_0 t + b_n \sin n w_0 t$$

$$= a_0 + \sum_{n=1}^{\infty} a_n \left[\frac{e^{jnw_0t} + e^{-jnw_0t}}{2}\right]$$

$$+ b_n \left[\frac{e^{jnw_0t} - e^{-jnw_0t}}{2j}\right]$$

$$= \underbrace{a_0}_{C_0} + \sum_{n=1}^{\infty} e^{jnw_0t} \underbrace{\left[\frac{a_n - jb_n}{2}\right]}_{C_n}$$

$$+ e^{-jnw_0t} \underbrace{\left[\frac{a_n + jb_n}{2}\right]}_{C_{-n}}$$

$$\boxed{-n = m}$$

$$= C_0 + \sum_{n=1}^{\infty} C_n e^{jnw_0t} + \sum_{n=1}^{\infty} C_{-n} e^{-jnw_0t}$$

$$= C_0 + \sum_{n=1}^{\infty} C_n e^{jnw_0t} + \sum_{m=-1}^{-\infty} C_m e^{jmw_0t}$$

$$\Rightarrow \boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jnw_0t}}$$

C_{-n} is the complex conjugate of C_n.

2 sided spectrum

$$C_0 = a_0$$

$$C_n = \frac{a_n - jb_n}{2}$$

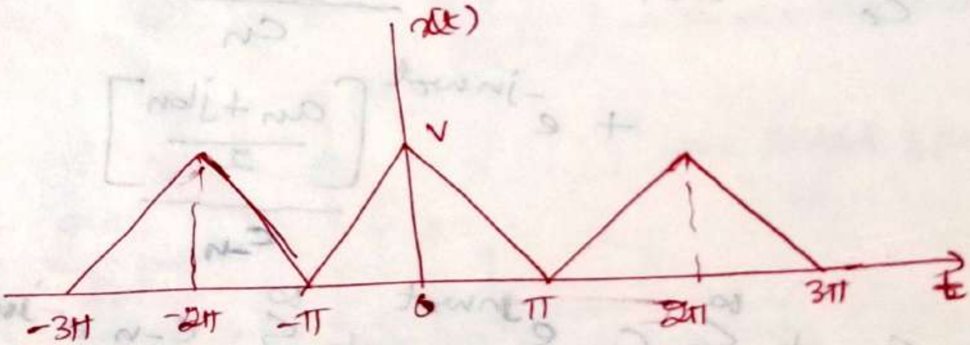
$$= \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jnw_0 t}\, dt$$

$$C_{-n} = \frac{a_n + jb_n}{2}$$

$$= \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{jnw_0 t}\, dt$$

Q11// Find the T. F. S. & E. F. S. expansion for the periodic signal shown in fig.

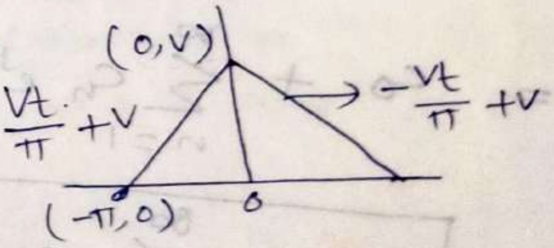
1)



ans:-

$T_0 = 2\pi$

$w_0 = 1$



$$x(t) = \begin{cases} \frac{Vt}{\pi} + V \;; & -\pi < t < 0 \\ -\frac{Vt}{\pi} + V \;; & 0 < t < \pi \end{cases}$$

Even

$b_n = 0$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

$$= \frac{\frac{1}{2}\times 2\pi \times V}{2\pi}$$

$$= \frac{V}{2}$$

(or)

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0}\left(\frac{Vt}{\pi}+V\right)dt + \int_0^{\pi}\left(-\frac{Vt}{\pi}+V\right)dt\right]$$

$$= \frac{1}{2\pi}\left[\frac{V}{\pi}\left[\frac{t^2}{2}\right]_{-\pi}^{0} + V[t]_{-\pi}^{0} - \frac{V}{\pi}\left[\frac{t^2}{2}\right]_0^{\pi} + V[t]_0^{\pi}\right]$$

⊛
$$= \frac{1}{2\pi}\left[\frac{V}{2\cancel{\pi}}\left(0-\pi^{\cancel{2}}\right) + V\pi - \frac{V}{2\cancel{\pi}}\pi^{\cancel{2}} + V\pi\right]$$

$$= \frac{1}{2\pi}\left[\frac{-V\pi}{2} + V\pi - \frac{V\pi}{2} + V\pi\right]$$

$$= \frac{1}{2\pi}\left[-V\pi + 2V\pi\right]$$

$$= \frac{1}{2\cancel{\pi}} \times V\cancel{\pi} = \frac{V}{2}$$

$$a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cos n\omega t\,dt$$

$$= \frac{4}{T_0}\int_0^{T_0/2} x(t)\cos n\omega_0 t\,dt$$

$$\boxed{\int_{-a}^{a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \quad \text{(for even function)}}$$

$$= \frac{\cancel{4}^{2}}{\cancel{2}\pi}\int_0^{\pi}\left(V - \frac{Vt}{\pi}\right)\cos nt\,dt$$

(area of odd signal is zero & even signal is 2 times the area)

$$= \frac{2V}{\pi} \int_0^{\pi} \underset{0}{\cancel{\cos nt}}\, dt - \frac{2V}{\pi^2} \int_0^{\pi} t \cos nt \, dt$$

$$= -\frac{2V}{\pi^2} \left[\frac{t \sin nt}{n} + \frac{\cos nt}{n^2} \right]_0^{\pi}$$

$$\boxed{\begin{aligned} &\sin n\pi = 0 \\ &\cos n\pi = (-1)^n \\ &e^{\pm jn\pi} = (-1)^n \end{aligned}}$$

$$\boxed{\begin{aligned} &\checkmark \int x \cos(ax)\, dx = \frac{\cos ax + ax \sin(ax)}{a^2} \\ &\int x \sin(ax)\, dx = \frac{\sin ax - ax \cos ax}{a^2} \end{aligned}}$$

$$= -\frac{2V}{\pi^2} \left[\frac{(-1)^n}{n^2} - \frac{1}{n^2} \right]$$

$$= \frac{2V}{\pi^2 n^2} \left[1 - (-1)^n \right]$$

$$= \begin{cases} 0 & ; n \text{ even} \\ \frac{4V}{\pi^2 n^2} & ; n \text{ odd} \end{cases}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$= \frac{V}{2} + \sum_{\substack{n=1 \\ (n=\text{odd})}}^{\infty} \frac{4V}{\pi^2 n^2} \cos n\hat{\omega_0 t}$$

$$= \frac{V}{2} + \frac{4V}{\pi^2} \cos t + \frac{4V}{9\pi^2} \cos 3t$$

$$+ \cdots$$

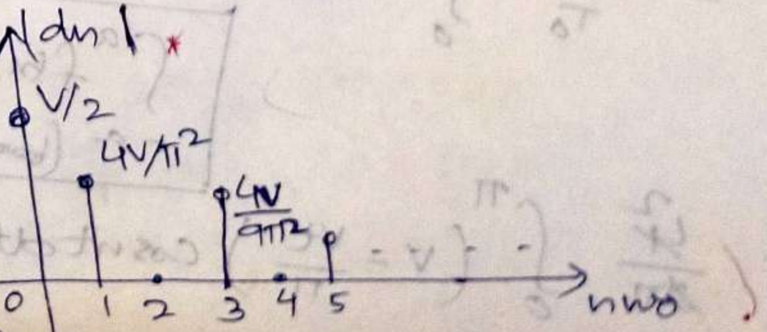
$$d_0 = a_0 = \frac{V}{2}$$

$$|d_n| = \sqrt{a_n^2 + b_n^2} \quad b_n \to 0$$

$$= |a_n|$$

$$= \frac{4V}{\pi^2 n^2} \text{ (odd } n)$$

$$x(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos(n\omega_0 t - \phi_n)$$



$$\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

$$= \tan^{-1}(0)$$

$$= 0$$

$$\Rightarrow x(t) = \frac{V}{2} + \sum_{n(\text{odd})}^{\infty} \frac{4V}{\pi^2 n^2} \cos nt$$

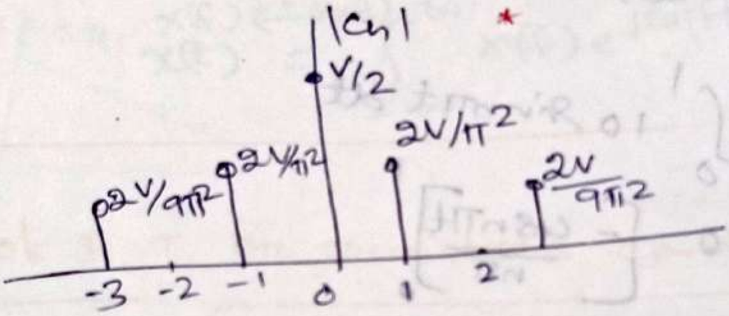
$C_0 = a_0 = \frac{V}{2}$

$C_n = \frac{a_n - j b_n}{2} = \frac{2V}{\pi^2 n^2}$ (odd n)

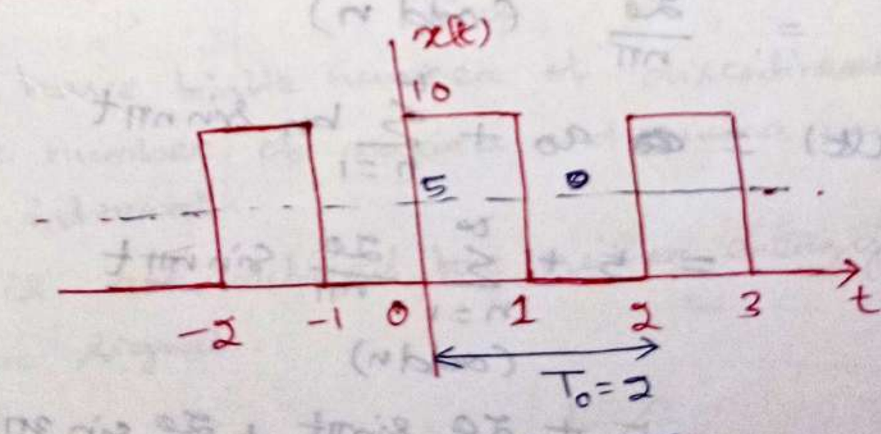
$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$= \sum_{\substack{n=-\infty \\ (\text{odd } n)}}^{\infty} \frac{2V}{\pi^2 n^2} e^{jn\omega_0 t}$$

$$= \cdots + \frac{2V}{\pi^2} e^{-jt} + \frac{V}{2} + \frac{2V}{\pi^2} e^{jt} + \cdots$$



① Q) A Periodic input signal x(t) is shown below. Find the T.F.S. representation.



ans:-

$\omega_0 = \pi$

hidden odd

$\downarrow$ $a_0 = ?$ $\downarrow$ $b_n = 0 ?$

$a_0 = \frac{(10)(1)}{2} = 5$ $\frac{\text{area}}{\text{fundamental period}}$

or $a_0 = \frac{1}{2}\int_0^1 10\,dt = \frac{10}{2} \times 1 = 5$

$$a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cos n\omega_0 t\, dt$$

$$= \frac{\cancel{2}}{\cancel{2}}\int_0^1 10\cos n\pi t\, dt$$

$$= 10\int_0^1 \cos n\pi t\, dt$$

$$= 10\left[\frac{\sin n\pi t}{n\pi}\right]_0^1$$

$$= 10 \times \left[\frac{\sin n\pi}{n\pi} - 0\right]$$

$$= 10 \times 0 = 0$$

$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t)\sin n\omega_0 t\, dt$$

$$= \frac{\cancel{2}}{\cancel{2}}\int_0^1 10\sin n\pi t\, dt$$

$$= 10\left[-\frac{\cos n\pi t}{n\pi}\right]_0^1$$

$$= \frac{10}{n\pi} \times \left[-(-1)^n + 1\right]$$

$$= \frac{20}{n\pi} \quad (\text{odd } n)$$

$$\therefore x(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin n\pi t$$

$$= 5 + \sum_{\substack{n=1 \\ (\text{odd } n)}}^{\infty} \frac{20}{n\pi}\sin n\pi t$$

$$= 5 + \frac{20}{\pi}\sin \pi t + \frac{20}{3\pi}\sin 3\pi t + \frac{20}{5\pi}\sin 5\pi t + \dots$$

Fouriere Transform

Fouriere Transform provides a frequency domain description of time domain signals and is extension of Fouriere series to non-periodic signals.

→ spectrum of F.T. is contineous whereas spectrum of F.S. is discrete.

$$x(t) \xleftrightarrow{F.T.} X(\omega)$$

F.T. of a signal $x(t)$, $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

$$X(\omega) \xleftrightarrow{I.F.T.} x(t)$$

I.F.T. $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$

$$x(t) \xleftrightarrow{F.T.} X(f)$$

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$X(f) \xleftrightarrow{I.F.T.} x(t)$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$$

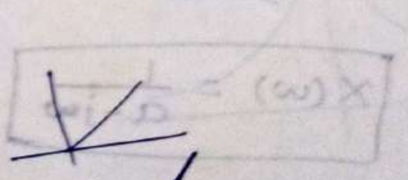
Convergence of F.T. or Dirichlet's conditions

(i) Fouriere Transform is defined for all energy & stable signals. i.e. $\int_{-\infty}^{+\infty} |x(t)|\, dt < \infty$

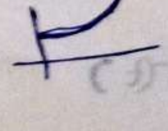
(ii) $x(t)$ have finite number of discontinuities and finite number of maxima and minima within any finite interval.

(iii) F.T. is not defined for neither energy nor power signal.

$x(t) = t\, u(t)$

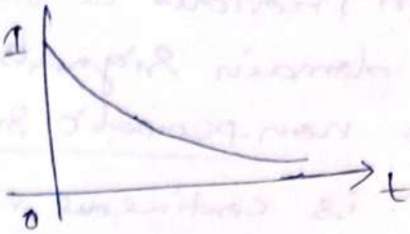


$x(t) = e^{2t} u(t)$



→ If signal is physically possible then F.T. possible.

① Q1 $x(t) = e^{-at} u(t)$ $a > 0$



$$X(w) = \int_0^{\infty} e^{-at} \cdot e^{-jwt} dt$$

$$= \int_0^{\infty} e^{-(a+jw)t} dt$$

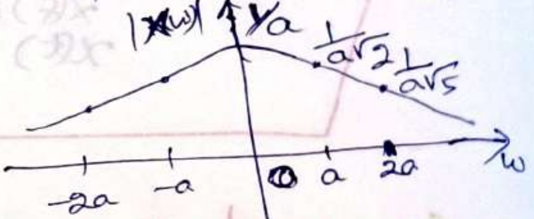
$$= \left[\frac{e^{-(a+jw)t}}{-(a+jw)} \right]_0^{\infty}$$

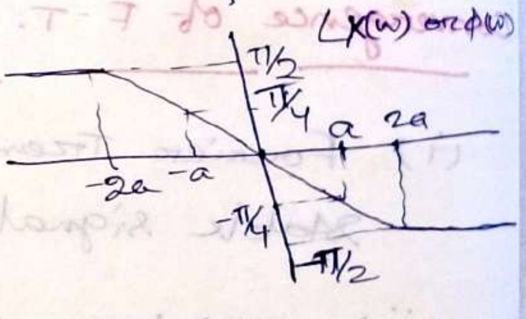
$$= \frac{1}{a+jw}$$

$$\boxed{e^{-at} u(t) \xleftrightarrow{F.T.} \frac{1}{a+jw}}$$

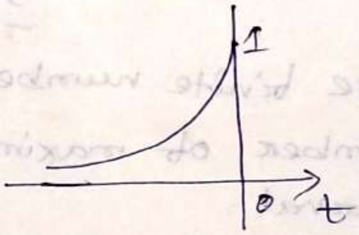
$$|X(w)| = \frac{1}{\sqrt{a^2+w^2}}$$

$$\angle X(w) = -\tan^{-1}\left(\frac{w}{a}\right) = \phi(w)$$





② Q1 $x(t) = e^{at} u(-t)$



$t \to w$

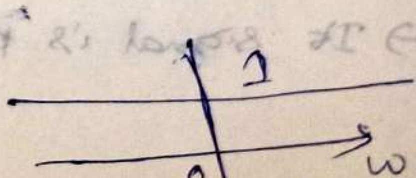
$-t \to -w$

$$\boxed{X(w) = \frac{1}{a-jw}}$$

③ Q1 $x(t) = \delta(t)$

$$X(w) = \int_{-\infty}^{\infty} \delta(t) e^{-jwt} dt$$

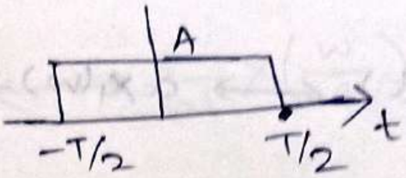
$$= 1$$



Signal contain all freq. compon
Same as white noise.

④ $x(t) = A\,\text{rect}(t/T)$

or $A\,\Pi(t/T)$



$$X(\omega) = \int_{-T/2}^{T/2} A e^{-j\omega t}\,dt$$

$$= A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T/2}^{T/2}$$

$$= \frac{2A}{2j\omega}\left[e^{j\omega T/2} - e^{-j\omega T/2}\right]$$

$$= \frac{2A}{\omega}\sin\left(\frac{\omega T}{2}\right)$$

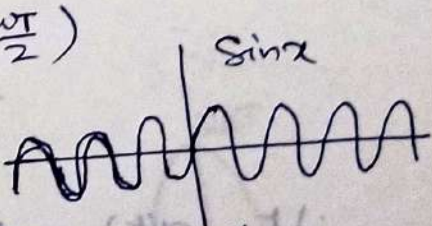
$$= \frac{2A\sin\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)\cdot\frac{2}{T}}$$

$\text{sinc}\,x = \frac{\sin \pi x}{\pi x}$ [illegible]

or $Sa(x) = \frac{\sin x}{x}$

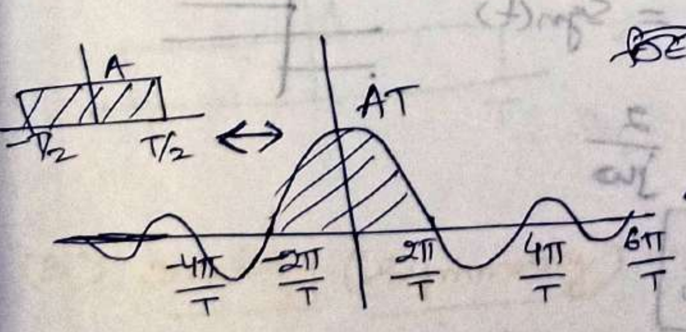
$=$ ~~$AT\,\text{sin}$ [illegible]~~ $= AT\,\text{sinc}\left(\frac{[illegible]}{2}\right)$

~~or $AT\,Sa$ [illegible]~~ or $AT\,Sa\left(\frac{\omega T}{2}\right)$



~~$f(x) = \pm1, \pm2, \pm3 \ldots$~~

~~[illegible] $\pm\frac{2\pi}{T}, \pm\frac{3\pi}{T}$, [illegible]~~



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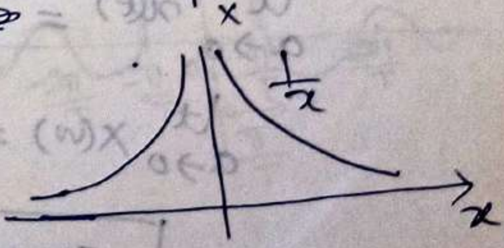


$$\frac{\omega T}{2} = \pm n\pi$$

$$\omega = \pm\frac{2n\pi}{T}$$ (zero box)

multiple of $2\pi = 0$

BW $= \infty$

Practical BW $= \frac{2\pi}{T}$, Spectral width $= \frac{4\pi}{T}$

Properties of fourier Transform

(a) linearity

$$a x_1(t) + b x_2(t) \longleftrightarrow a X_1(w) + b X_2(w)$$

Q1

$$x(t) = e^{-a|t|}$$

$$= e^{-at} \quad ; t > 0$$

$$= e^{at} \quad ; t < 0$$

$$x(t) = e^{-at} u(t) + e^{at} u(-t)$$

$$= \frac{1}{a+jw} + \frac{1}{a-jw}$$

$$= \frac{2a}{a^2+w^2}$$

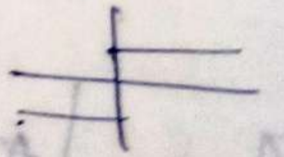
②

$$x(t) = e^{-at} u(t) - e^{at} u(-t)$$

$$X(w) = \frac{1}{a+jw} - \frac{1}{a-jw}$$

$$= \frac{-2jw}{a^2+w^2}$$

$$\lim_{a \to 0} x(t) = u(t) - u(-t) = \text{Sgn}(t)$$



$$\lim_{a \to 0} X(w) = \frac{-2jw}{w^2} = \frac{2}{jw}$$

$$\boxed{\text{Sgn}(t) \longleftrightarrow \frac{2}{jw}}$$

(2) Time scaling

$$x(t) \longleftrightarrow X(\omega)$$

$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

or $g(t) \longleftrightarrow G(f)$

$\Rightarrow g(at) \longleftrightarrow \frac{1}{|a|} G\left(\frac{f}{a}\right)$

$$x_1(t) = A\,\mathrm{rect}\left(\frac{t}{2T}\right)$$

$$= x(t/2)$$

$$a = \frac{1}{2}$$

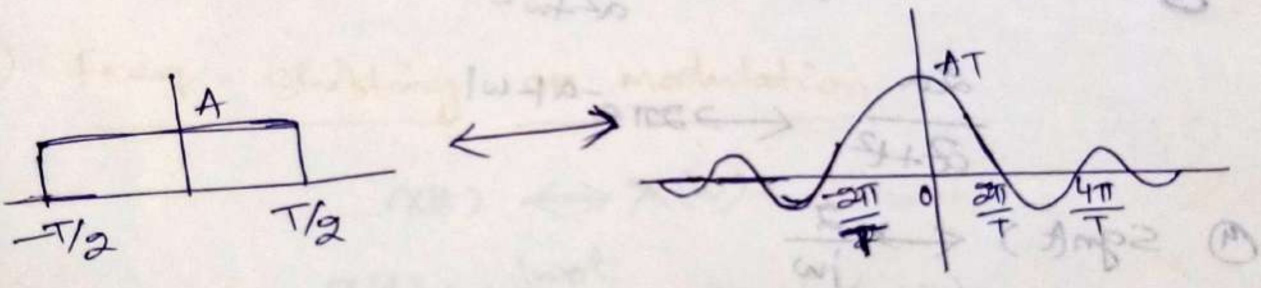
$\Big\downarrow$ F.T.

$x(t) \longleftrightarrow X(\omega)$

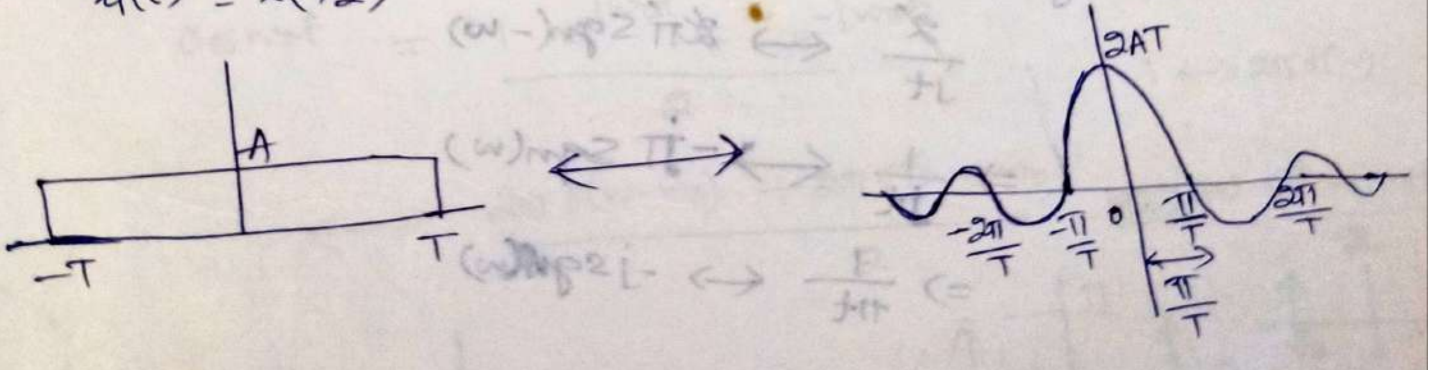
$A\,\mathrm{rect}(t/T) \longleftrightarrow AT\,\mathrm{sinc}\left(\frac{\omega T}{2}\right)$

$$X_1(\omega) = \frac{1}{\left|\frac{1}{2}\right|} X\left(\frac{\omega}{\frac{1}{2}}\right)$$

$$= 2X(2\omega) = 2AT\,\mathrm{sinc}\left(\frac{\cancel{2}\omega T}{\cancel{2}}\right)$$



$x_1(t) = x(t/2)$



(3) Duality (Symmetry):-

$$x(t) \longleftrightarrow X(\omega)$$

$$X(t) \longleftrightarrow 2\pi\, x(-\omega)$$

or $g(t) \longleftrightarrow G(f)$

$\Rightarrow G(t) \longleftrightarrow g(-f)$

$$2\pi x(t) = \int X(\omega)\, e^{j\omega t}\, d\omega$$

$$t \rightarrow -t$$

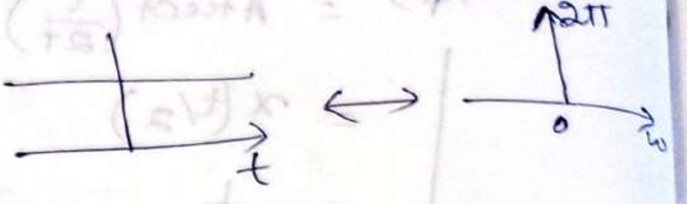
$2\pi x(-t) = \int X(w) e^{-jwt} dw$

$t \to w$

$2\pi x(-w) = \int X(t) e^{-jwt} dt$

① $\delta(t) \Leftrightarrow \underset{X(w)}{1}$

$\underset{X(t)}{1} \longleftrightarrow 2\pi \delta(w)$



② $e^{at} u(-t) = \frac{1}{a-jw}$

$\frac{1}{a-jt} \longleftrightarrow 2\pi e^{a(-w)} u(w)$

③ $e^{-a|t|} \longleftrightarrow \frac{2a}{a^2+w^2}$

$\frac{2a}{a^2+t^2} \longleftrightarrow 2\pi e^{-a|-w|}$

* ④ $sgn(t) \longleftrightarrow \frac{2}{jw}$

Duality,

$\frac{\cancel{2}}{jt} \longleftrightarrow \cancel{2}\pi\, sgn(-w)$

$\Rightarrow \frac{1}{jt} \longleftrightarrow -j\pi\, sgn(w)$

$\Rightarrow \frac{1}{\pi t} \longleftrightarrow -j\, sgn(w)$

⑤ $sgn(t) = 2u(t) - 1$

$u(t) = \frac{1+sgn(t)}{2}$

$u(t) \longleftrightarrow \frac{2\pi\delta(w) + \frac{2}{jw}}{2}$

$\boxed{u(t) \longleftrightarrow \frac{1}{jw} + \pi\delta(w)}$

④ Time Shifting :-

$$x(t) \Longleftrightarrow X(\omega)$$

$$x(t-t_0) \Longleftrightarrow e^{-j\omega t_0} \cdot X(\omega)$$

① $y(t) = e^{-2t} u(t-3)$

$$e^{-at} u(t) \rightarrow e^{-a(t-t_0)} u(t-t_0)$$

$$= e^{-2(t-3+3)} u(t-3)$$

$$= e^{-6} e^{-2(t-3)} u(t-3) \Longleftrightarrow e^{-6}\left[\frac{e^{-j\omega(3)}}{2+j\omega}\right]$$

② $Z(t) = \Pi\left(\frac{t-1}{2}\right)$

$$\Pi\left(\frac{t}{2}\right) \Longleftrightarrow 2\,\text{sinc}(\omega)$$

$$Z(\omega) = e^{-j\omega(1)} \cdot 2\,\text{sinc}(\omega)$$

$A = 1$
$T = 2$

⑤ freq. shifting or modulation :-

$$x(t) \Longleftrightarrow X(\omega)$$

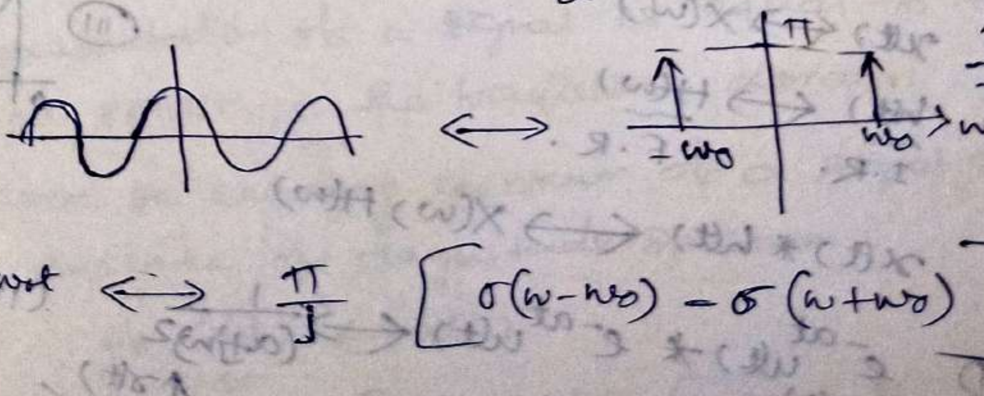
$$x(t) \cdot e^{j\omega_0 t} \Longleftrightarrow X(\omega - \omega_0)$$

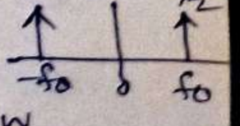
$$\cos\omega_0 t = \frac{1 \cdot e^{j\omega_0 t} + 1 \cdot e^{-j\omega_0 t}}{2}$$

$$1 \Longleftrightarrow 2\pi\,\sigma(\omega)$$

$$\Longleftrightarrow \frac{2\pi\,\sigma(\omega-\omega_0) + 2\pi\,\sigma(\omega+\omega_0)}{2}$$

(or) $\frac{\sigma(f-f_0)+\sigma(f+f_0)}{2}$





$$\sin\omega_0 t \Longleftrightarrow \frac{\pi}{j}\left[\sigma(\omega-\omega_0) - \sigma(\omega+\omega_0)\right]$$

⑥ Differentiation in time :-

$$x(t) \longleftrightarrow X(w) \qquad \frac{d\,x(t)}{dt} \longleftrightarrow jw\,X(w)$$

$$\frac{d}{dt} \longleftrightarrow jw$$

① $x(t) = \text{sgn}\,t = 2u(t) - 1$

$$\frac{d}{dt}\,x(t) = 2\delta(t)$$

$$\downarrow F.T.$$

$$jw\,X(w) = 2$$

$$\Rightarrow X(w) = \frac{2}{jw}$$

⑦ freq. Differentiation :-

$$x(t) \longleftrightarrow X(w)$$

$$-jt\,x(t) \longleftrightarrow \frac{d}{dw}X(w).$$

$$\boxed{\begin{array}{l} \frac{d}{dt} \longleftrightarrow jw \\ \frac{d}{dw} \longleftrightarrow -jt \end{array}}$$

① $y(t) = t\,e^{-at}u(t)$

$$= t\,x(t)$$

$$Y(w) = (-j)\,\frac{d}{dw}\left[\frac{1}{a+jw}\right]$$

$$= \frac{1}{(a+jw)^2}$$

⑧ convolution in time

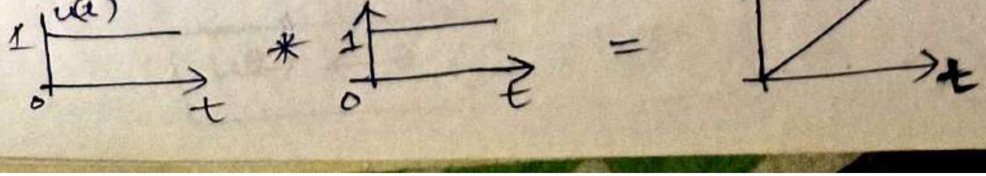
$$x(t) \longleftrightarrow X(w)$$

$$\underset{\text{I.R.}}{h(t)} \longleftrightarrow \underset{\text{F.R.}}{H(w)}$$

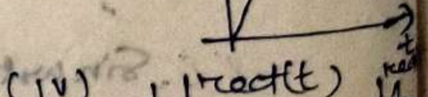
$$x(t) * h(t) \longleftrightarrow X(w)\,H(w)$$

① $e^{-at}u(t) * e^{-at}u(t) \longleftrightarrow \frac{1}{(a+jw)^2}$

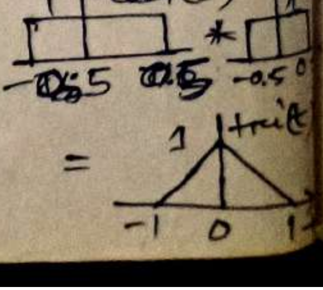
⑪



(iii)



(iv)



⑨ Freq. convolution :-

$x_1(t) \longleftrightarrow X_1(\omega)$ and $x_2(t) \longleftrightarrow X_2(\omega)$

then $x_1(t) \cdot x_2(t) \longleftrightarrow \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega) \right]$

① $x(t)\cos\omega_0 t \longleftrightarrow \frac{1}{2\pi} X(\omega) * \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$

$$\longleftrightarrow \frac{X(\omega-\omega_0) + X(\omega+\omega_0)}{2}$$

Convolution

$x(t) * \delta(t-t_0) = x(t \pm t_0$

Product

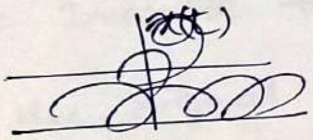
$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

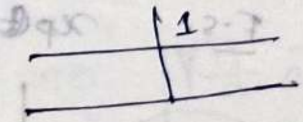
Integ. $\int x(t)\,\delta(t-t_0) = x(t_0)$

⑩ Integration in time

$\int_{-\infty}^{t} x(\tau)\,d\tau \longleftrightarrow \frac{X(\omega)}{j\omega}$ if $X(0) = 0$

$\longleftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0)\,\delta(\omega)$ if $X(0) \neq 0$





① $x(t) = \delta(t)$

$$\underbrace{\int_{-\infty}^{t} \delta(\tau)\,d\tau}_{u(t)} = \frac{1}{j\omega} + \pi(1)\,\delta(\omega)$$

⑪ Rayleigh's Energy theorem or Parseval's power theorem

It states that the energy or power in the time domain representation of a signal is equal to the energy or power in the frequency domain.

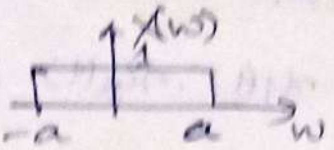
→ The power or energy spectrum of a signal is defined as the square of magnitude spectrum.

If $x(t) \longleftrightarrow X(\omega)$

$$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \underbrace{|X(\omega)|^2}_{\text{E.S.D or P.S.D.}}\,d\omega$$

Q// Find the energy in the signal $x(t) = \frac{\sin at}{\pi t}$

$$\frac{\sin at}{\pi t} \longleftrightarrow$$



$$x(t) = \frac{\sin at}{\pi t}$$

$$E_{x(t)} = \frac{1}{2\pi} \int_{-a}^{a} (1)^2 \, dw$$

$$= \frac{a}{\pi}$$

F.T. of Periodic signal

F.S: $x_p(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jnw_0t}$

It is already freq. domain

$\downarrow$ F.T.

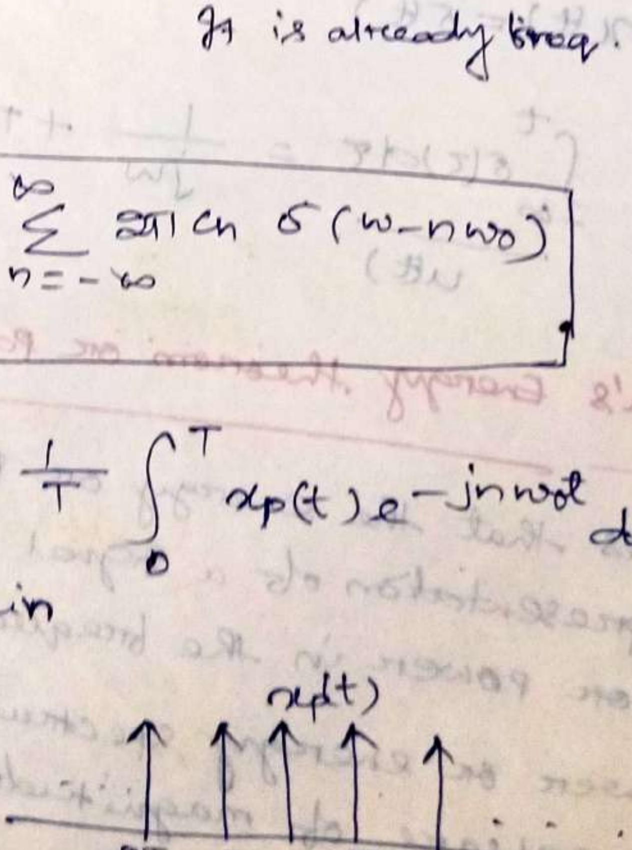
$$\boxed{X_p(w) = \sum_{n=-\infty}^{\infty} 2\pi c_n \, \delta(w - nw_0)}$$

$1 \longleftrightarrow 2\pi\delta(w)$

$1 \cdot e^{jw_0t} \longleftrightarrow 2\pi\delta(w-w_0)$

$$c_n = \frac{1}{T} \int_0^T x_p(t) e^{-jnw_0t} \, dt$$

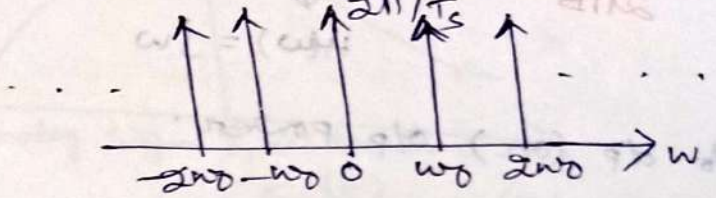
Ex:- Impulse train



$$C_n = \frac{1}{T_s}\int_0^{T_s} \delta(t) \cdot e^{-jn\omega_0 t}\, dt$$

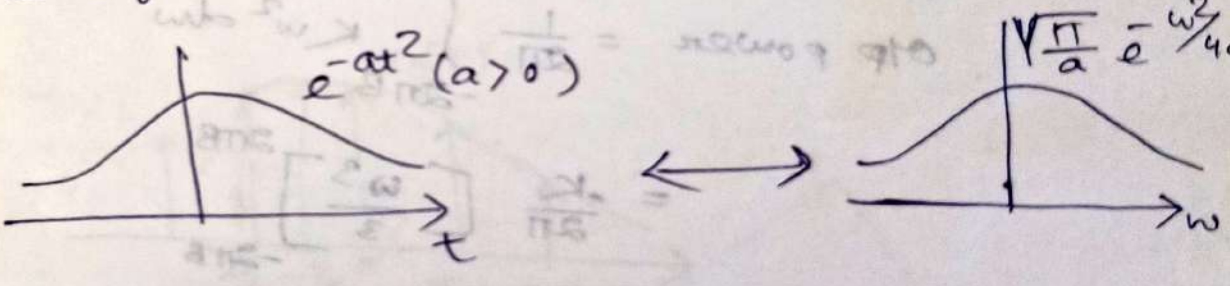
$$= \frac{1}{T_s}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} \frac{2\pi}{T_s}\,\delta(\omega - n\omega_0) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} \delta(f - nf_0)$$



$$\boxed{2\pi\delta(\omega) = \delta(f)}$$

→ For Impulse train & gaussian pulse, the F.T. of the signal is itself (or) same.



→ Sinc & impulse when convolve itself, then the same signal we get.

* $y(t) = x(t) * h(t)$ (before F.T. of periodic signal)

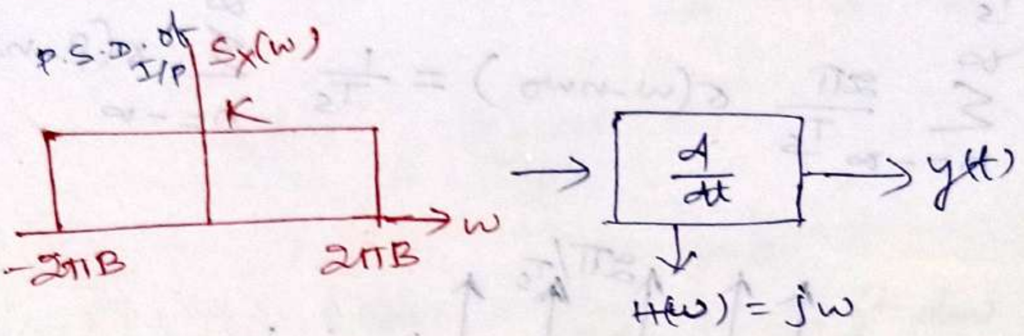
$\downarrow$ F.T.

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

$$|Y(\omega)|^2 = |X(\omega)|^2 |H(\omega)|^2$$

$$\boxed{\underbrace{S_y(\omega)}_{\text{o/p P.S.D}} = \underbrace{S_x(\omega)}_{\text{I/p P.S.D.}} |H(\omega)|^2}$$

Q1) A power signal x(t) whose PSD is shown in fig. is applied to an ideal differentiator, find the mean square value of the output of the differentiator.



m.s.v. of o/p (or) o/p power.

$$S_y(w) = S_x(w)|H(w)|^2$$

$$\Rightarrow S_y(w) = K|jw|^2 \quad ; \quad -2\pi B < w < 2\pi B$$

↓

o/p P.S.D

$$\text{o/p power} = \frac{1}{2\pi}\int_{-2\pi B}^{2\pi B} K w^2 \, dw$$

$$= \frac{K}{2\pi}\left[\frac{w^3}{3}\right]_{-2\pi B}^{2\pi B}$$

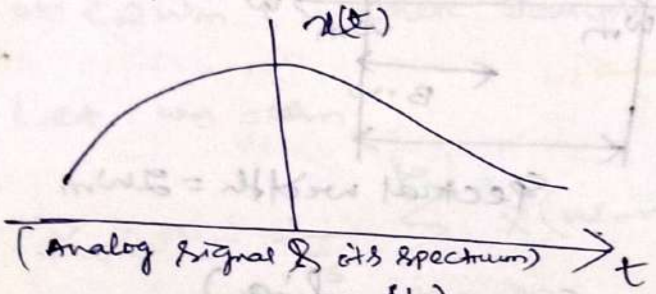
$$= \frac{K}{6\pi}\left[8\pi^3B^3 + 8\pi^3B^3\right)$$

$$= \frac{K}{\underset{3}{\cancel{6\pi}}} \times \overset{8}{\cancel{16}}\pi^3 B^3$$

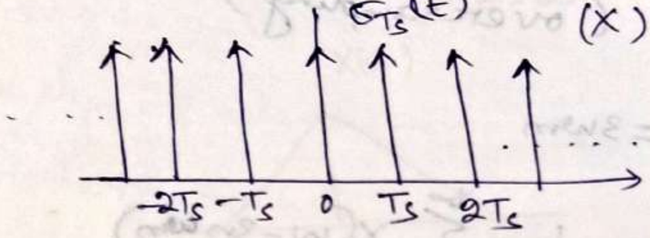
$$= \frac{8}{3}K\pi^2 B^3$$

Sampling theorem :- ~~As continuous time~~ The original signal can be completely reconstructed ~~[illegible]~~ in the receiver when the sampling freq. ~~[illegible]~~ twice of the max^m signal freq. i.e. $f_s \geq 2f_m$.

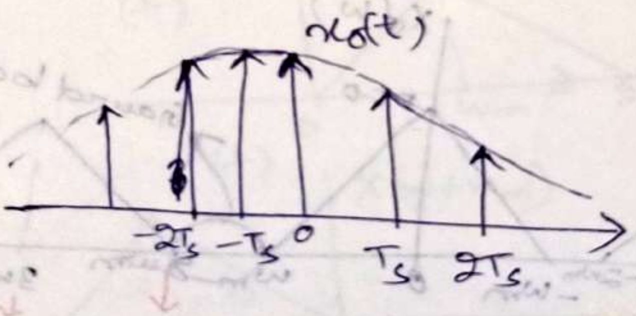
Ideal Sampling :- described a sampled signal as a weighted sum of impulses, the weights being equal to the values of the analog signal at the impulse locations.



(Analog signal & its spectrum)



(sampling function & it's spectrum)



⑧

$x_\delta(t) \rightarrow$ Ideally sampled signal.

$$x_\delta(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

$$\downarrow \text{F.T.}$$

~~$X_\delta(\omega)$~~ $x_1(t) \cdot x_2(t) \longleftrightarrow \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right]$

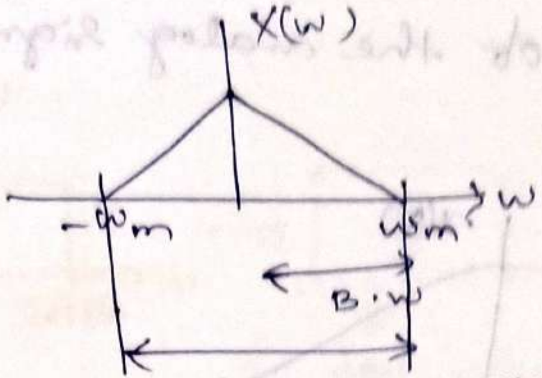
$$X_\delta(\omega) = \frac{1}{2\pi}\left[X(\omega) * \frac{2\pi}{T_s}\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)\right)$$

$$\boxed{X_\delta(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega-n\omega_0)}$$

~~ω_0~~ → sampling rate

Assume a bandlimited spectrum.

$$x(t) \longleftrightarrow X(w)$$



Spectral width = $2w_m$

<u>Case-1</u> $w_o > 2w_m$ (oversampling)

Let $w_o = 3w_m$.

$$X_\delta(w) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(w-3n w_m)$$

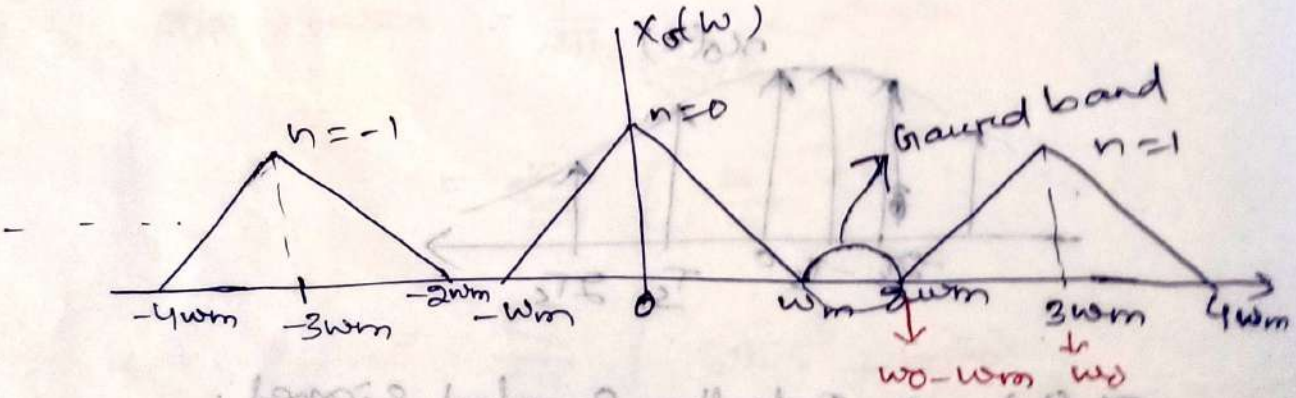


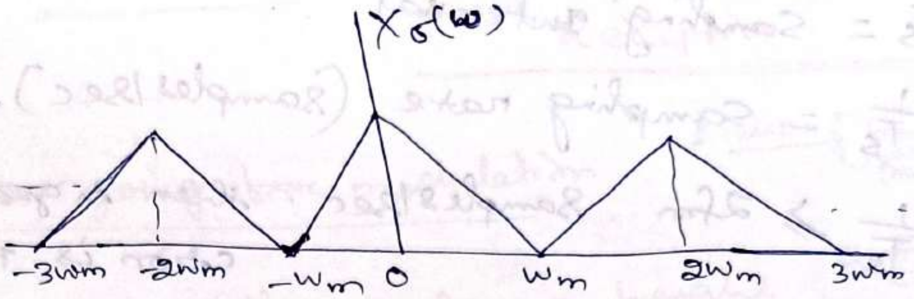
Image don't overlap,

when $w_o - w_m > w_m$

$\Rightarrow w_o > 2w_m$

<u>Case-II</u> $w_o = 2w_m$ (Critical Sampling)

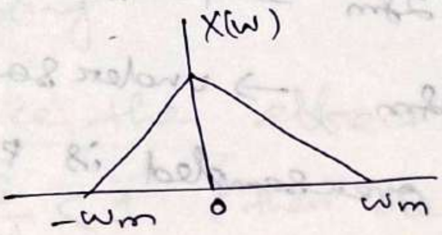
$$X_\delta(w) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(w-2n w_m)$$



Case-III $\omega_0 < 2\omega_m$ (Under Sampling)

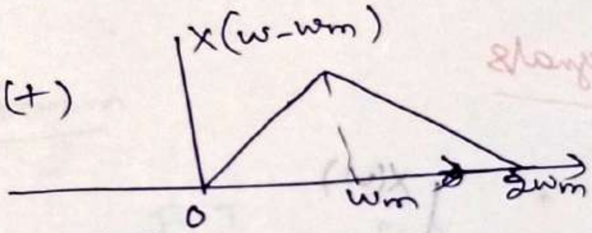
Let $\omega_0 = \omega_m$

$$X_\sigma(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_m)$$

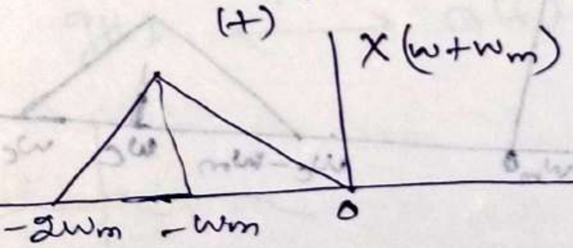


(+)

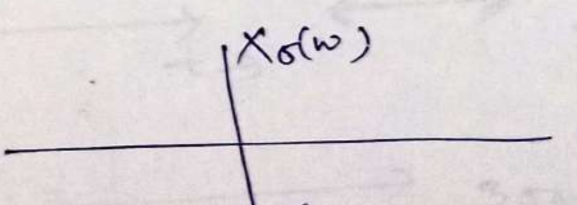
(ii)



(+)



=>



const. (dc.) Spectrum

Aliasing
(or)
(Spectral folding)

To avoid aliasing, $\omega_0 \geq 2\omega_m$

min (ω_0) = Nyquist rate = $2\omega_m$

$$\boxed{\frac{1}{T_s} = f_s = 2f_m}$$

T_s = Sampling Interval

$\frac{1}{T_s}$ = sampling rate (samples/sec).

$\frac{1}{T_s} \geqslant 2f_m$ samples/sec. then signal reconst
ction is possible.

$\frac{1}{T_s} < 2f_m$ → Aliasing effect (signal reconstruction is not possible)

$\frac{1}{T_s} = 2f_m$ → Nyquist rate

↓

It is the minimum sampling rate.

when $\frac{1}{T_s} > 2f_m$ samples/sec → over sampled.

$\frac{1}{T_s} < 2f_m$ → under sampled.

→ Practically always oversampled is preferred.

Band pass signals